DOI:
10.2478/v10286-012-0016-6

Management
2012
Vol.16, No. 1
ISSN 1429-9321

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An evaluation of project completion with application of fuzzy set theory

1. Introduction

Many cases of projects indicate that fewer than half of projects met cost and schedule targets (e.g. Nitithamyong, Skibniewski 2006; Robertson, Williams 2006; Shore 2008). The increment of chance for a project completion with the success, i.e. according to original specification, may be considered in terms of allowing in planning stage the critical factors of a project, and then a continuous project monitoring. Planning should concern the execution of project tasks and take into consideration a complexity of project, project manager's skill and a form of data description (e.g. fuzzy data).

A project execution depends on many factors, such as the kind of project, accessibility of resources, project management, and external environment (np. Robertson, Williams 2006). In the research works of project management field, among the factors of a project success may encounter (Spałek 2004): proper planning and control of project, executive support, customer involvement, competent staff and experienced project manager, clear business objectives and firm basic requirements.

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The features of project management impose to take account of some methods concerning an uncertainty modelling. If the uncertainty derives from the randomness, then for its description may be used the probability theory. In turn, the uncertainty derives from the partial information (its fuzzy form), there is a possibility to used the fuzzy set theory.

For more complete description of the variables occurring in project management field, it seems suitable to describe the data in an accurate and fuzzy way in single approach. For instance, the project time required by the client and the constraints concerning an order of project tasks may be described in an accurate way. In turn, for instance the planned time of task execution may be described in fuzzy form. The application of fuzzy set theory allows linking the numeric information with linguistic information (gained from experts). It seems very important in case of unique tasks of project for which do not exist the proper set of data for estimation of variable distribution and then using e.g. program evaluation and review technique (PERT).

In case of decision problems, where the knowledge for their modeling is acquired from the experts, a horizontal form of fuzzy set description is often used. This approach describes the fuzzy sets using so-called α-cut of a set. Considering fuzzy sets in terms of α-cut facilitates identification of a membership function according to a knowledge acquisition method from experts (Piegat 2003).

In the paper, the application of fuzzy set theory concerns a fuzzy number in term of α-cut for the task duration, and the verification which part of a fuzzy number fulfils the assumed constraints. The part of a fuzzy number for critical tasks that fulfils the assumed constraints may be interpreted as the level of certainty for a project completion. The presented approach is the continuation of the research in field of a project prototyping problem (alternative variants of a project completion), that is described in (Relich 2011).

The paper is organised as follows. A fuzzy constraint satisfaction problem for the estimation of a project completion is formulated in section 2. A method for obtaining a fuzzy rule base is shown in section 3. An example of the approach, which presents a using of constraint programming techniques for implementation of fuzzy constraint satisfaction problem for project task duration, is presented in section 4.

2. Fuzzy constraint satisfaction problem for project evaluation

The resources of an enterprise characterise its model. A project model derives from client's requirements or from the needs the enterprise, in case of an own project. In the enterprise and project model same parameters are chosen, among which the sets of decision variables and constraints may be distinguished. In case of an enterprise, the decision variables contain its resources (e.g. number of employees, financial means). In turn, for a project the decision variables concern e.g. time and cost of project task execution. The project implementation follows according to kind of the enterprise and its resources. For this reason, the fields of enterprise activity influence on the fields of project management. For instance, the type of enterprise activity determines the feasibility of a project. Also size or type of organisation determines a project execution.

In general, a model specification involves a declaration of the sets of decision variables, their domains, and constraints that imposed on subsets of variables. In this context, it seems natural to classify some decision problems as Constraints Satisfaction Problem (CSP). A considered approach of specification, determined by constraints of reference model of decision problem, enables a simplified description of actuality, i.e. a description encompasses the assumptions of object, implementing therein tasks, and a set of routine queries (the instances of decision problems) that in framework of CSP are formulated.

In a classical form, the structure of constraints satisfaction problem is described as follows (Rossi 2000): $CSP = ((V, D), C)$, where: V – a set of variables, D – a set of discrete domains of variables, C – a set of constraints. Taking into consideration the imprecise characteristics of project management, it is assumed Fuzzy Constraints Satisfaction Problem (FCSP) as follows:

$$FCSP = ((\hat{V}, D), C)$$

where:

$\hat{V} = \{\hat{v}_1, \hat{v}_2, ..., \hat{v}_n\}$ – a finite set of n fuzzy variables,

$D = \{d_1, d_2, ..., d_n\}$ – a set of domains for n variables that in form of fuzzy number (a finite set of discrete α-cut) are described,

$C = \{c_1, c_2, ..., c_m\}$ – a finite set of m constraints limiting and linking decision variables.

The set of variables may contain a number of financial resources (e.g. cash, deposits, short-term payments) in time unit, a number of working hours for a group of employees (e.g. managers, designers, workers) in time unit, a description of project activities. The project activity is specified by the starting time (s_j) and the duration (t_j) of j-th activity.

An exemplary constraint, which link the fields of the enterprise's management with the project management, may be following: the financial means for a project cannot be greater than total financial means in the enterprise in time unit. The project constraints concern the relations between activities (order constraints) and a horizon of project completion.

A fuzzy constraint C can be characterized by the logic value $E(C)$, $E(C)$ Î [0,1]. In turn, value $E(C)$ allow to determine the level of certainty (CF) of reference model's constraints satisfaction, i.e. a kind of certainty threshold. For instance, $CF = 1$ means the all constraints hold, and $CF = 0,8$ means that they are almost satisfied. The level CF is defined due to the formula $CF = \min\{E(C)\}$ (Bocewicz et al. 2009, p. 107).

The assumed model enables descriptive approach to the problem statement, encompasses constraint satisfaction problem structure and then allows implementing the considered problem in the constraint programming environment. The idea behind the proposed approach assumes the considered system can be represented in terms of a knowledge base. Knowledge base comprises of facts and rules determining the system's properties and relations linking them respectively.

Knowledge base can be specified in terms of a system (Bubnicki 1998). At the input of the system are the variables regarding the fundamental attributes of the object that are known and given by the user. In the considered knowledge base for the enterprise-project model, there are, for example, variables concerning the number of an enterprise's resources or the project structure. The output of the system is described by the attributes of the object that are unknown or are only partially known. In the considered case, there can be included variables regarding e.g. the cost or time of activity, or usage of resources.

Classification of the decision variables in knowledge base as input-output variables is arbitrarily made and permits to formulate two classes of standard queries, in a forward and in a backward way for considered problem. In case of the forward way, the considered problem regards the answer to the following question: does there exist a schedule meeting assumed constraints, and if so, what are its parameters? The determination of fuzzy rule base for duration of project task is presented in next section.

3. Determination of fuzzy rules

The planning issue and then the successive monitoring of the project, is one of the most important elements of project management that determines its success or failure (Kerzner 2009; Szyjewski 2004; Trocki et al. 2009). So, there is a need to develop method that will enable an early detection of discrepancies in a project execution. Moreover, the method should determine the alternative variants that meet the goal of the project and avoid the estimated discrepancies (Relich 2011).

The precision of estimation for project execution depends on an uniqueness of project activities. The features of project management impose a using of some methods for uncertainty modelling. If the uncertainty stems from randomness, then for modelling can be used probability theory. In turn, if the uncertainty stems from an information incompleteness (human ignorance), then the uncertainty can be described by fuzzy logic theory.

In order of an entire description of the uncertainty in project management, it seems natural to combine these two, above-mentioned methods into an approach. The approach should take into account an accurate as well as an uncertain specification of variables. Exemplary, in precise form can be described the required by client time horizon of project and order of constraints. In fuzzy form can be described e.g. time of an activity completion.

The determination of fuzzy data rule begin from the assumption concerning a number and shape of membership function for fuzzy sets that describe the decision variable. In the next stage, there are calculated a membership for each fuzzy set. The learning data is assigned into set where is the maximal membership. In this way, each data determines single rule. If the rules are contradictory then for the each rule is assigned, so called degree of accuracy, and to select one rule with maximal value of degree. An entire description of above-mentioned procedure for obtaining fuzzy data rule is presented in D. Rutkowska et al. (1999, pp. 117-119).

In the considered approach, the knowledge base (also rules base) is described in terms of CSP, and a verification of its completeness is connected with guaranteeing coherence of knowledge base. A coherence verification of knowledge base follows through seeking rules that cover all values of input variables, and proofing whether for these values there is explicitly possible obtaining the values for output variables. If knowledge base enables an answer to user queries, then in knowledge base must exist a set of constraints that contain the relationships linking the variables from queries with these from knowledge base (Bocewicz et al. 2009, pp. 157-158). In turn, a verification of continuity of inference chain enables a checking of knowledge base coherence in terms of CSP, and it guarantees also completeness and consistency of knowledge base. Implementing the logic-algebraic method can also detect the redundancy (Bubnicki 1999). An entire description of inference, as well as the proof for completeness and consistency of knowledge base is presented in G. Bocewicz et al. (2009, pp. 95-104).

The proposed approach enables linking the numeric and linguistic information that derives from the experts. There is also possibility to specify the variables in an accurate (as singleton) and fuzzy way. Thus, the duration of project activities as imprecise (fuzzy) data may be treated.

Imprecise variables (e.g. time of an activity $\hat{t}_j$) specified by fuzzy sets and determined by membership function can be characterized by the α-cut. Also, the standard algebraic operations to the fuzzy numbers may be defined in form of the α-cut, $\alpha \in [0,1]$ (Bocewicz et al. 2009, p. 120; Łęski 2008, p. 66). Above-described determination of fuzzy data rule follows for each *j*-th activity, for which is no possibility for a precise time specification.

For fuzzy activity time $\hat{t}_j$ and other constraints (e.g. concerning an order of tasks and deadline *H*), there is sought a schedule for consider project (the fuzzy beginning moment of *j*-th activity $\hat{s}_j$) with the certainty threshold *CF*. Thus, the discrete values α-cut for the beginning moment of *j*-th activity are searched.

A concept of constraint programming (CP) software technology can be considered as a pertinent framework for declarative description CSP and then development of decision support system. The main idea behind the CP concept is based on subsequent phases of constraint propagation and variable distribution. A CSP can always be solved with brute force search. All possible values of all variables are enumerated and each is checked to see whether it is a solution. However, for many intractable problems, the number of candidates is usually too large to enumerate them all. CP has developed some ways (constraint propagation and variable distribution) to solve CSPs that greatly reduce the amount of search needed. This is sufficient to solve many practical problems (e.g. further considered scheduling). CP is qualitatively different from the other programming paradigms, in terms of declarative, object-oriented, and concurrent programming. Compared to these paradigms, constraint programming is much closer to the ideal of declarative programming: to say *what* we want without saying *how* to achieve it (Van Roy and Haridi 2004, p. 749). CP offers a more flexible modelling framework than mathematical programming (Hooker 2002, p. 295). A significant feature of the above-described reference model is its description in a declarative form that provides a base for making a task-oriented decision support system. For those reasons, CP is chosen to describe a CSP instead of linear (integer) programming. In turn, applications of fuzzy set theory in project scheduling show that most of the research on project scheduling has been focused on fuzzy PERT and fuzzy CPM (Dubois et al. 2003).

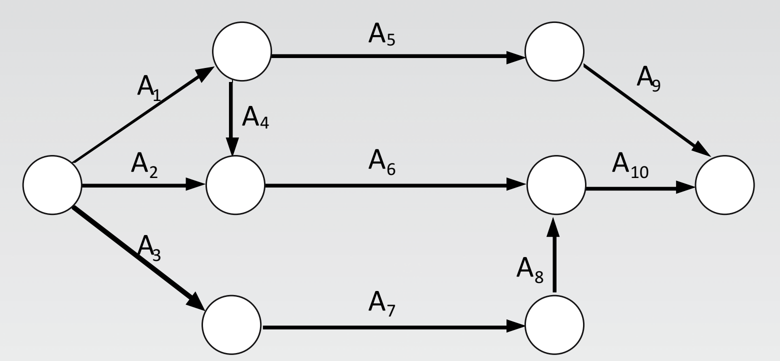
The implementation of considered approach in form of constraint programming languages imposes some assumptions that are related e.g. with discrete domains of decision variables. This approach is similar to fuzzy set theory, where some discrete linguistic information are given by the experts. For this reason, it is further assumed that the possibility of routine queries formulation and then searching their solutions, is determined be following assumptions:

- a set of whole numbers for decision variables,
- a description of decision variables in form of discrete α-cuts,
- a using of fuzzy arithmetic operator for the description of relation between fuzzy sets.

4. Example

The example aims to illustrate a possibility of *RCSP* specification for the decision problem of project completion in assumed time. The considered approach has been implemented in constraint programming technique Mozart Oz (www.mozart-oz.org). The activity network diagram for considered project $P = \{A_1, ..., A_{10}\}$ is presented in figure 1.

Figure 1. Activity network of project

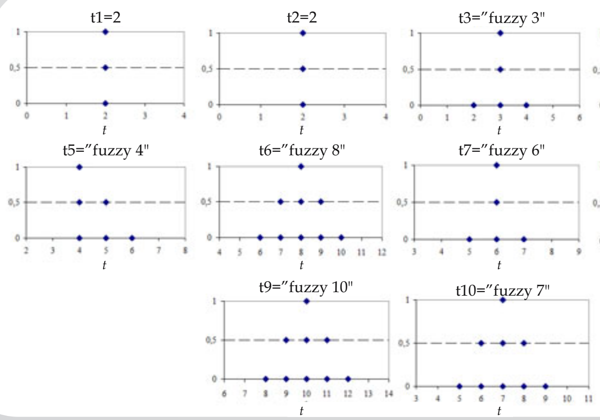


Source: own study

The order of the constraints, according to the activity network of the project, are following: C_1: $s_4 \geq s_1 + t_1$, C_2: $s_5 \geq s_1 + t_1$, C_3: $s_6 \geq s_4 + t_4$, C_4: $s_6 \geq s_2 + t_2$, C_5: $s_7 \geq s_3 + t_3$, C_6: $s_8 \geq s_7 + t_7$, C_7: $s_9 \geq s_5 + t_5$, C_8: $s_{10} \geq s_6 + t_6$, C_9: $s_{10} \geq s_8 + t_8$.

The fuzzy durations of *j*-th activities $\hat{t}_j$ are determined with using the fuzzy rule base. It is assumed that fuzzy durations of *j*-th activities are described by three the discrete α-cuts. The illustration of the fuzzy duration for project activities is presented in figure 2.

Figure 2. Fuzzy duration of activities



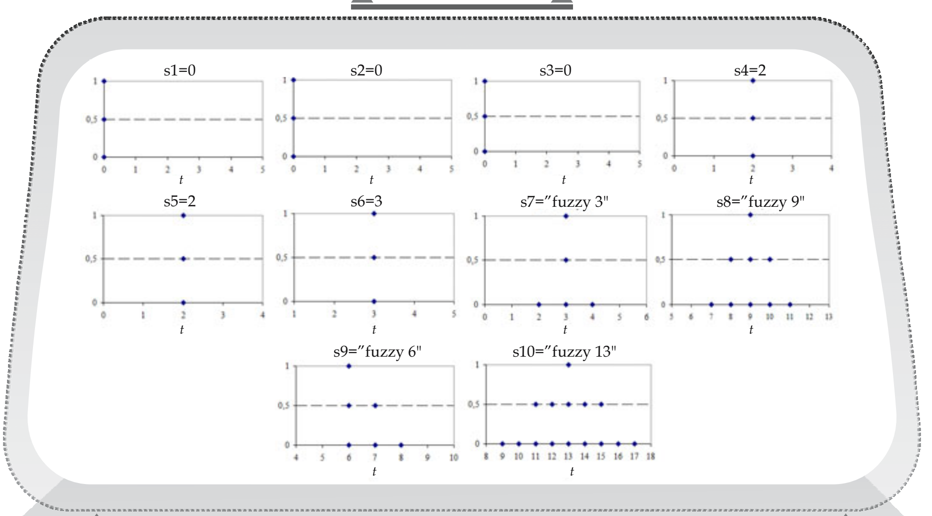
Source: own study

It is assumed that time horizon equals 23 time units (H = {0,1, ..., 23}) and expected level of certainty $CF \geq 0{,}8$. The problem is reduced to the following question: is there, and if so, what form does a plan have that completion time does not exceed the deadline *H*, and that fulfils the required value of *CF*?

The answer to the above-formulated question is related with determination of fuzzy moments of activity beginning $\hat{s}_j$. This results a seeking of variability concerning the sets and then the shape of fuzzy numbers that describes the moments of activity beginning. Thus, the considered problem is reduced to the following question: are there, and if so, what form do the membership functions of beginning moment have that completion time does not exceed the deadline *H*, and that fulfils the required value of *CF*?

An exemplary admissible solution is presented in figure 3. The moments of activity beginning in α-cuts form are described, and show in what time (interval) each activity may start. Note that the solution contains singletons, e.g. $\hat{s}_1$ and $\hat{s}_2$ as well as fuzzy form of numbers, e.g. $\hat{s}_0$.

Figure 3. Fuzzy time of activity beginning



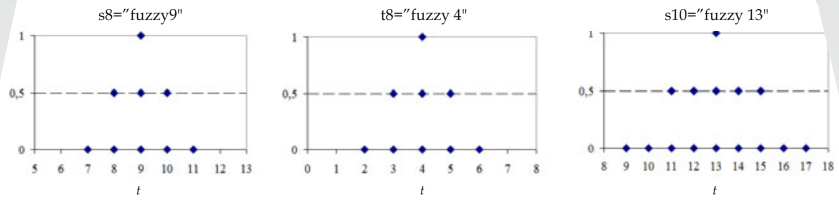
Source: own study

The determination of fuzzy beginning moment (in the discrete α-cut) for activity A_{10} is presented in figure 4. The entire description of standard algebraic operations in fuzzy form as well as rules for the plan of project completion is presented in G. Bocewicz et al. (2009, pp. 108-126).

A solution, presented in figure 3, is one of the admissible solutions for considered decision problem. The plan concerning activity execution for the admissible solution is presented in figure 5.

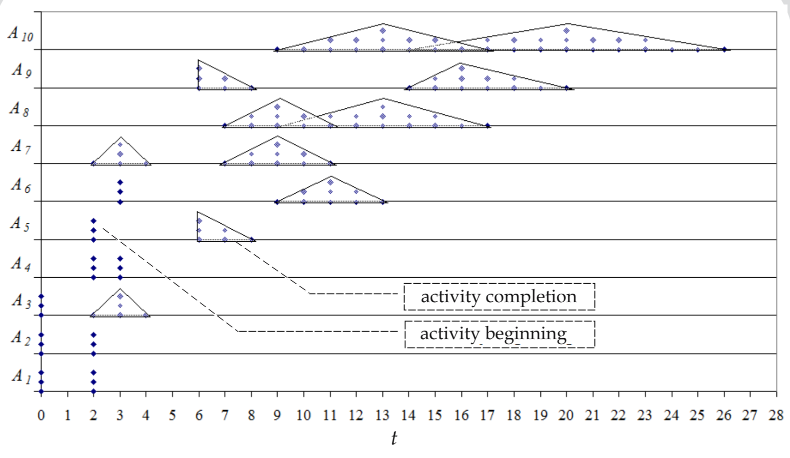
An activity is described through two fuzzy numbers. First indicates the activity beginning, second – activity completion. The assumed certainty threshold

Figure 4. Sum of two fuzzy numbers: $s_8 + t_8 = s_{10}$



Source: own study

Figure 5. Plan of project completion for certainty threshold $\geq 0{,}8$



Source: own study

$CF \geq 0,8$ means that in worst case 80% values of fuzzy numbers (discrete values of α-cut) fulfil the constraints concerning the reference model. In presented example, the duration of activity A_{10} contains in 85,7% (18 from 21 discrete values of α-cut) in project horizon $H \leq 23$.

The presented example illustrates the possibility of decision problem formulation that contains variables in imprecise form. The example concerns the determination of values for decision variables that guarantee a fulfilment of assumed constraints. If for the assumed constraints, there are not solutions then there is still possibility to reformulate the decision problem into backwards (reverse) way (Relich 2011). An exemplary routine query can be as follows: is there, and if so, what form should have an activity time ($\hat{t}_j$) to do not exceed the deadline H, and that fulfils the required certainty threshold CF.

5. Conclusions

The increase of a demand for new knowledge, that enables solution of the problems during the complex project execution, follows in a contemporary enterprise. In this case, the knowledge concerning project management has the particular significance. Especially, the identification of project success or failure as well as proper project planning and monitoring is desirable, what is usually connected with specific methods and techniques. In case of projects carried out on a client order, erroneous estimation of expenditures and project deadlines may result penalties being accrued, as agreed upon in the contract or covering the costs with the company's own money. A wrong decision may worsen the liquidity of an enterprise or even lead to its bankruptcy. In this situation, it seems extremely important to evaluate a project completion in assumed deadline.

A model of enterprise activity and project management may be specified through the sets of decision variables, their domains, and constraints that imposed on subsets of variables. The assumed model enables descriptive approach to the problem statement, encompasses constraint satisfaction problem structure and then allows implementing the considered problem in the constraint programming environment. The idea behind the proposed approach assumes the considered system can be represented in terms of a knowledge base. Knowledge base comprises of facts and rules determining the system's properties and relations linking them respectively. The proposed approach assumes a type of model encompassing open structure enabling to take into account different sorts of variables and constraints as well as to formulate forward and backward decision problem.

The proposed extension of presented model can compete with the common used solutions concerning project evaluation methods, such as critical path method (CPM), PERT, and variation of them (e.g. CPM/COST). It enables seeking an answer to the following two classes of standard routine queries: "what results from premises?" (forward reasoning) and "what implies conclusion?" (backward reasoning). Moreover, the model contains distinct and imprecise (fuzzy) data, in a unified way. It seems very important in case of projects with the unique activities, for which do not exist an appropriate set of data concerning the completed projects, and the estimation of activity duration is usually based on linguistic information from experts. Thus, the imprecise form of an activity description implies an application of fuzzy set theory.

A constraint can be treated as a logical relation among several variables, each one taking a value in a given domain. To solve such a problem stated by the set of constraints that specify a problem at hand, the concept of constraint programming is employed. Constraint programming is an emergent software technology for declarative description of constraint satisfaction problem and can be considered as a pertinent framework for development of decision support system software aims. The main idea behind the constraint programming concept is based on subsequent phases of constraint propagation and variable distribution.

The advantages of the proposed approach include the possibility of the description of enterprise and project management in terms of single knowledge base. Moreover, in the presented approach it is possible to determine a certainty threshold for a project completion. A successive project monitoring enables e.g. analysis of variation for a certainty threshold, aimed at supporting project manager by a decision concerning a rejection of project as well as gaining additional resources to complete the project.

Summary

An evaluation of project completion with application of fuzzy set theory

The project management contains such elements as management of time, cost, communications, procurement, quality, risk or scope of project. Each of these fields can be considered as a set of constraints, and then there is a possibility to verify their fulfillment in sense of an enterprise's constraints and its environment. These constraints determine a completion of project activities and its success or failure, finally. The paper aims to present a problem of project management in terms of fuzzy constraints satisfaction problem, and then the using of constraint programming techniques to the evaluation of project completion. A fuzzy constraints satisfaction problem enables a description of data in distinct, as well as imprecise form, in a unified framework. It seems especially important in case of unique activities of project, when their estimation is based on linguistic information from experts.

Streszczenie

Ocena realizacji przedsięwzięcia z wykorzystaniem teorii zbiorów rozmytych

W ramach zarządzania przedsięwzięciem wymienia się najczęściej takie elementy jak zarządzanie czasem, kosztami, komunikacją, dostawami, jakością, ryzykiem czy zakresem projektu. Każdy z tych obszarów zarządzania projektem można rozpatrywać w postaci zbioru ograniczeń, a następnie sprawdzać ich spełnienie w aspekcie ograniczeń wynikających z charakteru przedsiębiorstwa wdrażającego przedsięwzięcie oraz jego otoczenia. Ograniczenia te determinują realizację poszczególnych czynności projektu i ostatecznie to czy zakończy się on sukcesem, czy niepowodzeniem. Celem pracy jest przedstawienie problemu zarządzania przedsięwzięciem w postaci rozmytego problemu spełniania ograniczeń, a następnie wykorzystanie technik programowania z ograniczeniami do oceny realizacji przedsięwzięcia. Rozmyty problem spełniania ograniczeń umożliwia wyrażenie danych tak w postaci precyzyjnej, jak i nieprecyzyjnej w ramach jednego podejścia. Wydaje się to szczególnie istotne w przypadku planowania realizacji czynności unikalnych projektu, gdy ich szacowanie jest dokonywane przede wszystkim w oparciu o opinie ekspertów.

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